MATHEMATICAL MODELLING OF WIND TURBINES

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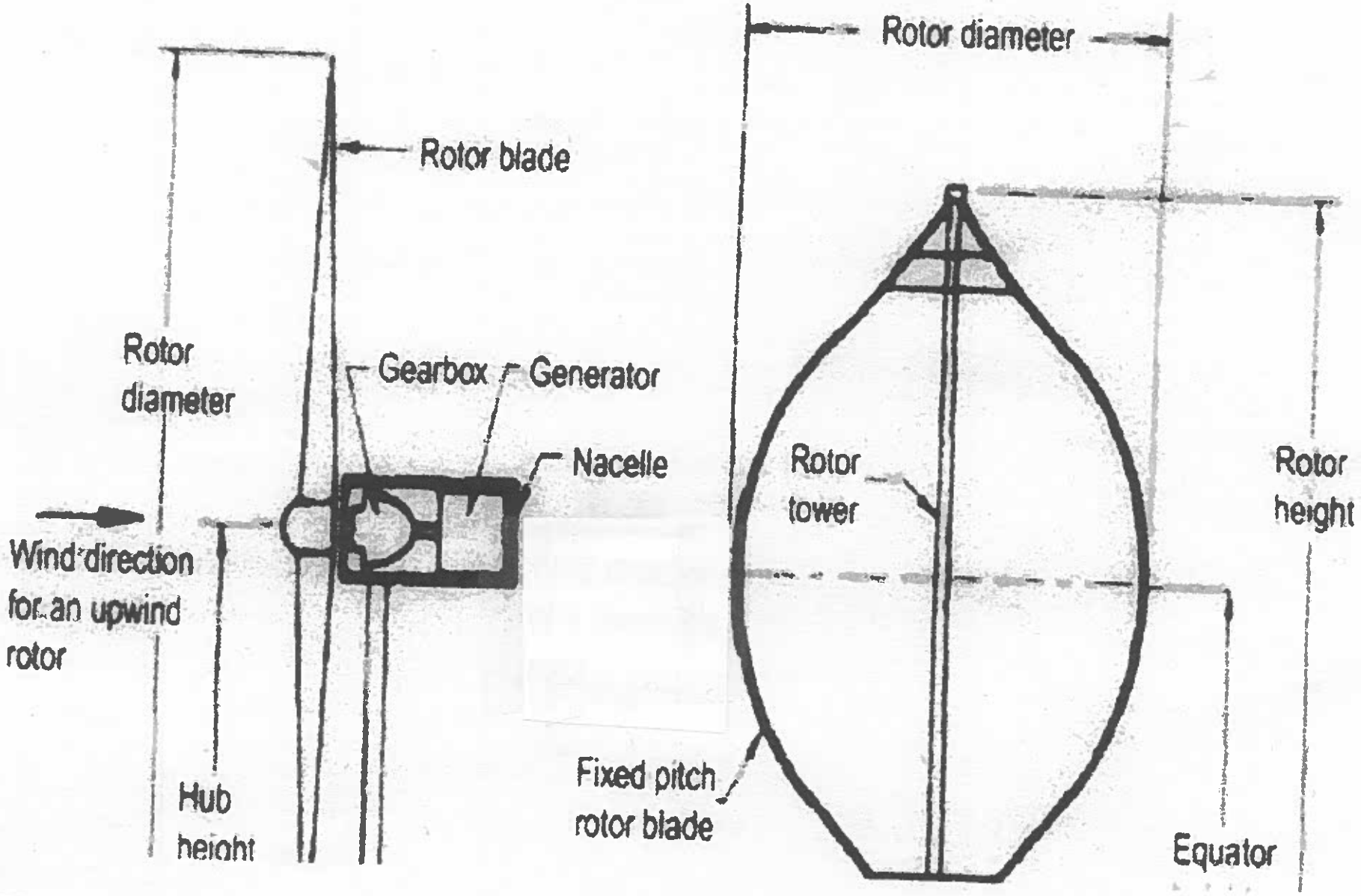
GRADUATE MODELLING CAMP

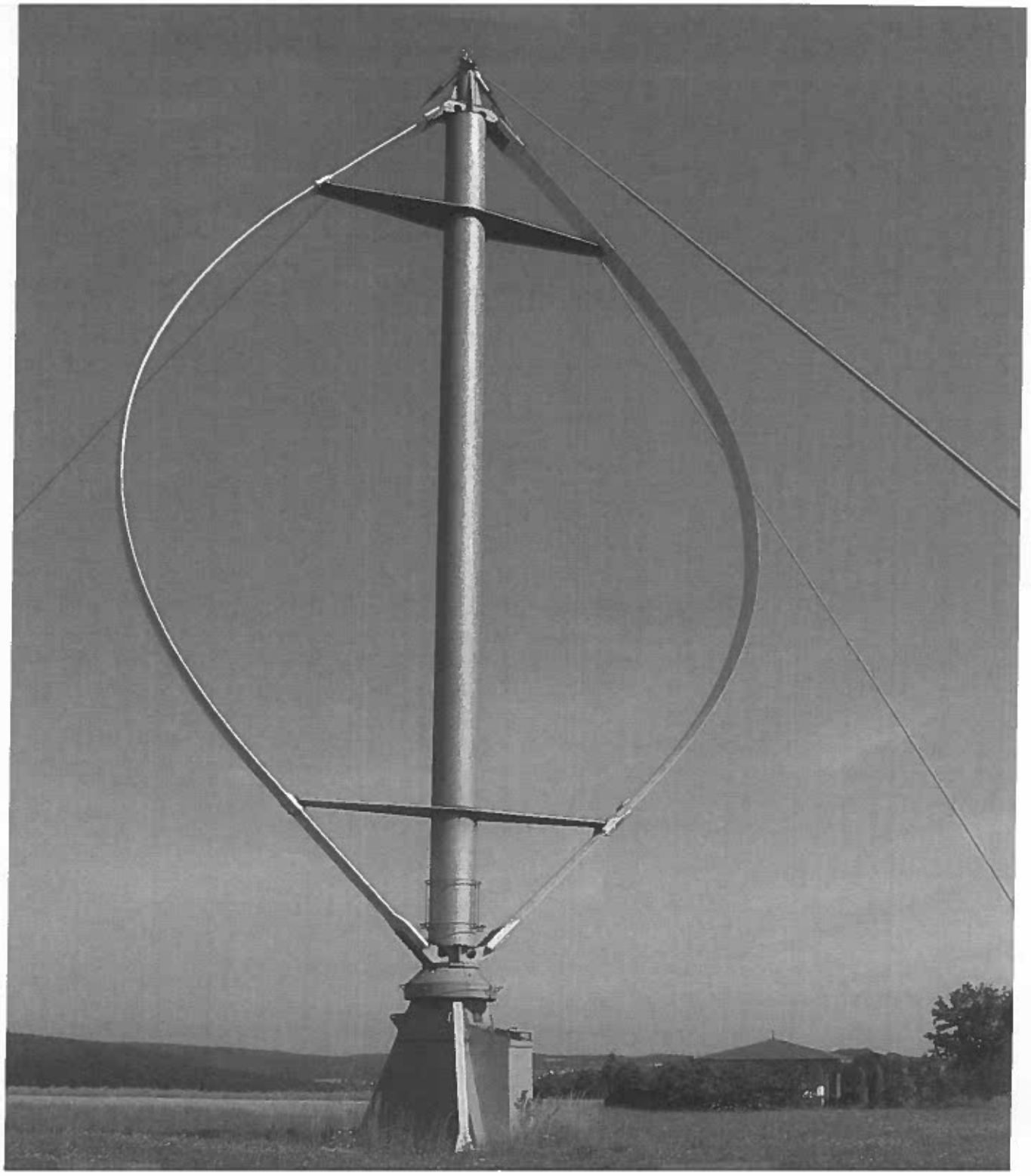

Rotor blade
Rotor diameter
Gearbox
Generator
Nacelle
Wind direction for an upwind rotor
Hub height
Rotor diameter
Rotor tower
Rotor height
Fixed pitch rotor blade
Equator

HAWT

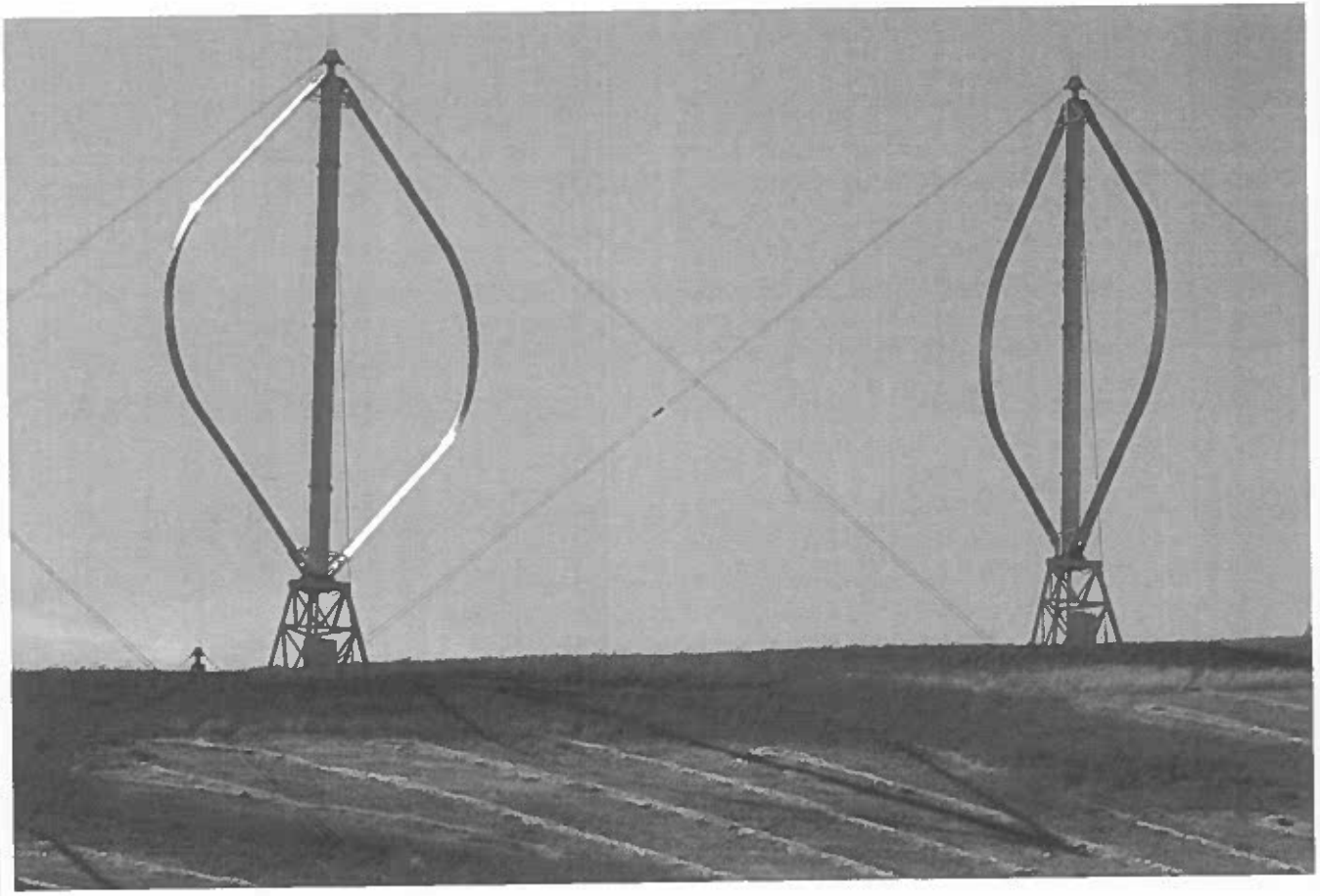
VAWT

HORIZONTAL AXIS WIND TURBINE

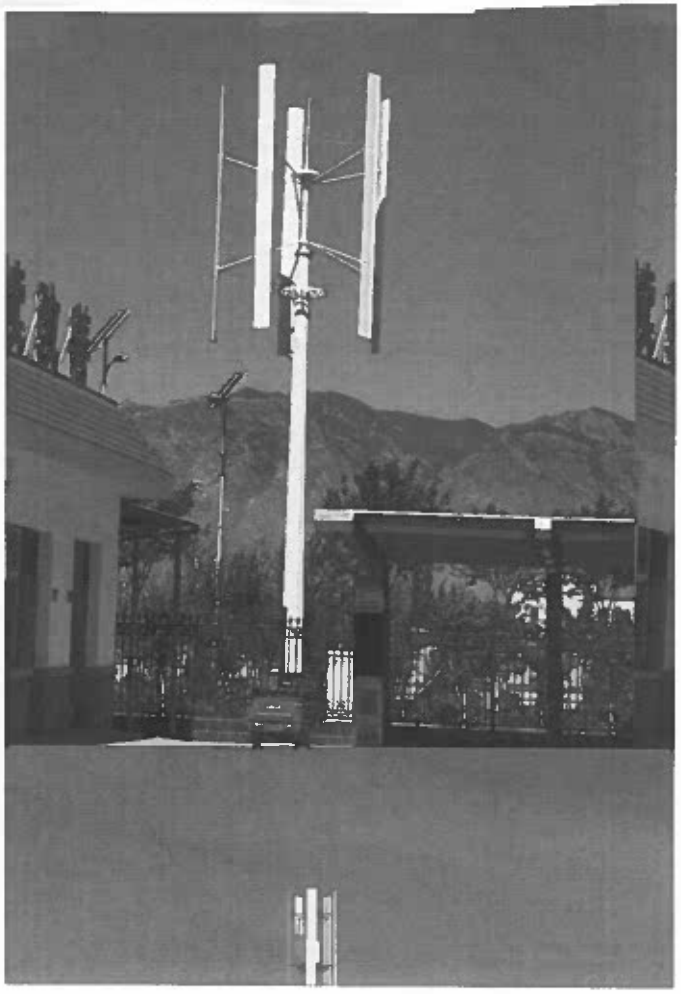
VAWT



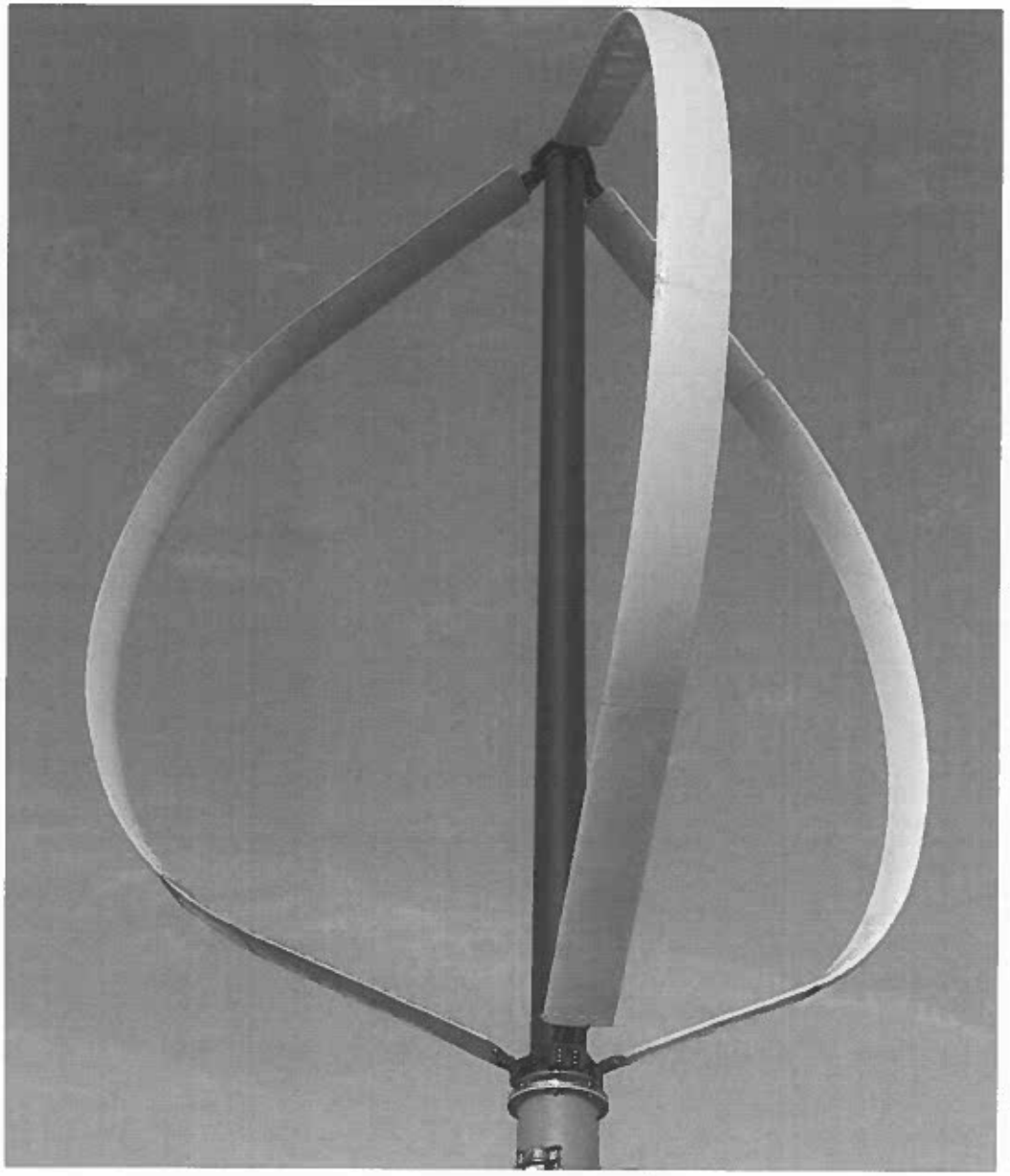
VERTICAL AXIS WIND TURBINES



Though it has been hailed by some as the future of wind, the vertical-axis wind turbine has yet to make a mark in the growing wind market. Photo: Dreamstime.com



Vertical Axis Wind Turbine



VAWT

VAWT – Vertical Axis Wind Turbines

- Savonius is another type of VAWT
- It is a vertical axis turbine since its axis of rotation is vertical to the ground.
- It can self-start
- Have not been very successful commercially
- Operates primarily on drag principle

HORIZONTAL AXIS WIND TURBINE

- EFFICIENCY
  - ACTUATOR DISK MODEL
  - BETZ LIMIT
- FULL WAKE BEHIND WIND TURBINE
  - CONSERVED QUANTITIES
  - SIMILARITY SOLUTION
- TURBULENT FAR WAKE
  - PRANDTL'S MIXING LENGTH MODEL EDDY VISCOSITY
  - CONSERVED QUANTITIES
  - SIMILARITY SOLUTION
  - BOUNDARY OF WAKE

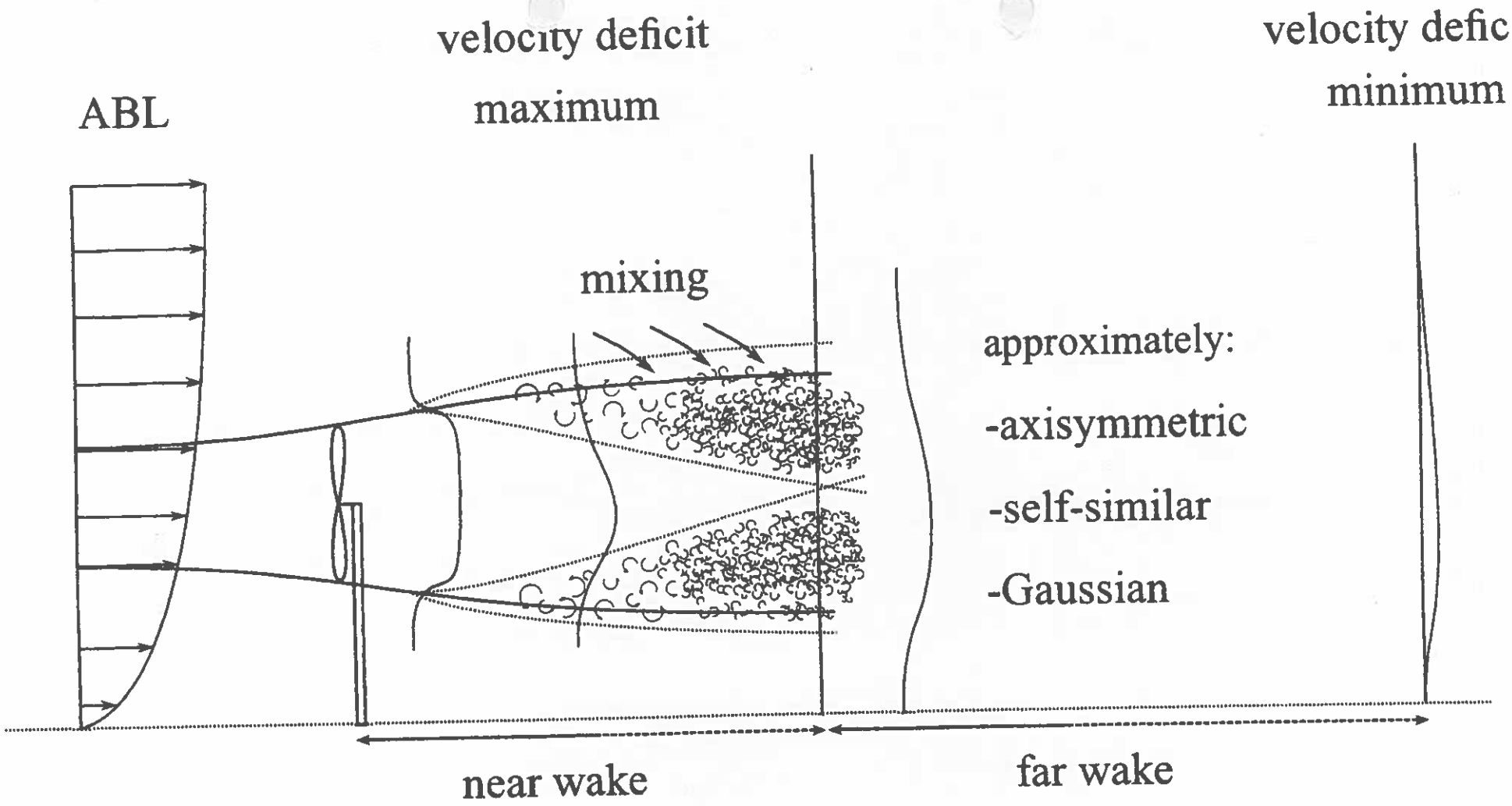


Figure 6: Velocity profile in the wake of a wind turbine.

Wake Effects of Wind Turbines

Photo: Aeolus

HAWT STRESS IN ROTATING BLADES

STRESS IN ROTATING RECTANGULAR BEAM

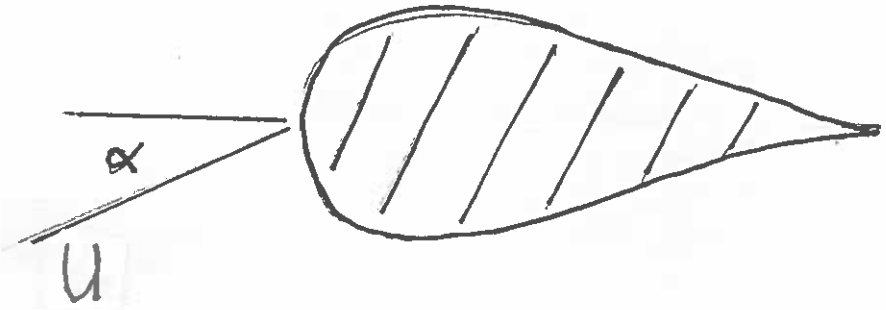
- PLANE STRESS THEORY OF ELASTICITY
- ROTATING AXES THEOREM
- AIRY STRESS FUNCTION FOR NON-ZERO BODY FORCE
- GENERALISED BIHARMONIC EQUATION
- POLYNOMIAL SOLUTION
- PRINCIPAL STRESSES $\tau_1 > \tau_2 > \tau_3$

  MAXIMUM NORMAL STRESS AT P $= \tau_1$

  MAXIMUM SHEARING STRESS AT P $= \frac{1}{2}|\tau_1 - \tau_3|$

VAWT AND HAWT

- LIFT ON BLADE



THIN SYMMETRICAL AEROFOIL

ANGLE OF ATTACK = α

COMPLEX POTENTIAL $W(z) = \phi + i\psi$

BLASIUS THEOREM FOR LIFT AND DRAG

$$F_x - i F_y = \frac{1}{2} i \rho \oint \left(\frac{dw}{dz}\right)^2 dz$$

KUTTA - JOUKOWSKI LIFT THEOREM

DRAG $F_x = 0$ LIFT $F_y = -\rho U \Gamma$

CIRCULATION $\Gamma = \oint_C \underline{v} . d\underline{x}$

STARTING VORTEX AND GENERATION OF CIRCULATION Γ

AEROFOIL EQUATION

$$\Gamma = \int_0^L \gamma(\xi)\, d\xi \quad : \qquad \frac{1}{2\pi} \int_0^L \frac{\gamma(\xi)\, d\xi}{\xi - x} = -\alpha U$$
